Stephanie Wright

What Is Randomness?

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Introduction

The concept of randomness is central to many areas of scientific and mathematical study. Despite the wide application of this concept to scientific experiment and explanation, no generally accepted definition of randomness has yet been developed. The difficulty of constructing a theory of randomness can be attributed in part to epistemological problems. The subjective experience of unpredictability is closely connected to our understandings of randomness, but it is not a sufficient condition for a definition, since a non-random event may be experienced as random if an observer is constrained by epistemic limitations. The requirement, proposed by Antony Eagle, of an intersubjective consensus on unpredictability in order to define an event as random, lessens but does not resolve this difficulty. A definition of randomness as unpredictability is therefore useful in considering the effects and applications of concepts of randomness, but problematic if we are seeking to explain randomness in terms of possible ontological properties.

In spite of this, a theory that attempts to define randomness in terms of its intrinsic properties rather than its observed effects is clearly necessary if randomness is to be seen as a feature of reality, and not merely as a useful conceptual tool. This does not require that a theory of randomness must apply to every sense in which we use the word. Rather, it is possible to distinguish between varying uses of the term 'random' in differing contexts. Firstly, the qualities of subjective experience which contribute to a belief that an event is 'random' may be analysed. Secondly, the ways in which apparently random events might be demonstrated to have been produced by non-random processes can be explored. A consideration of the relationship between scientific knowledge and perceptions of randomness provides insight into the sorts of epistemic problems entailed by ideas of randomness, and the extent to which we are capable, or might in the future be capable, of overcoming these problems. Finally, a consideration of events which, upon examination, consistently display the properties of randomness as defined by subjective experience, may allow us to speculate on the possibility of the existence of ontic randomness.

1. Subjective Randomness

The subjective experience of randomness in empirical observation is intimately to connected to our notions of what randomness is. Although the subjective experience of randomness incorporates a wide range of notions such as a “lack of order, periodicity, pattern, aim, or purpose”,[1] Deborah Burnett has proposed that a definition of randomness according to everyday experience should encompass two main features: unpredictability and fairness.[2] However, it is unclear as to why Burnett includes the notion of fairness in her definition of randomness, since while it appears that all random processes should be fair, not all fair processes are random (choosing a student to present in class in order of surname, for example). The idea of fairness as suggested by Burnett seems to refer merely to the independence of random outcomes from human influence, and thus appears to be a property that is related to, and contingent upon, unpredictability. The fairness of a coin toss, for example, is predicated upon the presumed inability of humans to influence or predict the outcome; that is, upon the unpredictability of the outcome.

Unpredictability seems therefore to be the common underlying feature of our experience of randomness. When we subjectively describe any experienced event as 'random', although our exact meaning may incorporate any combination of the above properties and possibly more, what we actually mean is that we did not foresee, and cannot imagine how we could have predicted, the occurrence of that event. A unique event might be described as random because the subject was unable to predict its occurrence by observing the events and conditions that preceded it. A sequence of events might be perceived as random because a subject is unable to predict subsequent events on the basis of preceding ones. Similarly, a process might be seen as random because a subject is unable to predict the outcomes generated by it. In any case, the subjective experience of unpredictability appears to be closely related to wider understandings of randomness.

2. Randomness and Unpredictability: Epistemic Difficulties

The connection between the experience of unpredictability and concepts of randomness raises an epistemic problem. The claim 'Event X is unpredictable' is an ontological statement, inasmuch as it assigns the property of 'unpredictability' to the event itself. At the same time, it may be seen as the expression of an epistemic fact – that of our own inability to predict event X. The

1 Deborah Bennett, 'Defining Randomness' in *Philosophy of Statistics* edited by Prasanta S. Bandyopadhyay and Malcom R. Forster (Oxford: Elsevier, 2011), 633.

2 Ibid.

extent to which this ontological claim can be considered separately from the epistemic fact of our inability to predict the outcome represents a significant problem for a definition of randomness. On the one hand, if an event were ontologically unpredictable, this would always result in the inability of a subject to make predictions about it. Ontological unpredictability thus entails epistemic unpredictability.

The converse, on the other hand, is not necessarily true. If a series of events were genuinely random, we would expect that this would be reflected in the inability of the subject to predict the sequence by perceiving a pattern in the data; conversely, if a sequence were non-random, we would expect that a subject might be able to identify a pattern in the data and make accurate predictions. However, psychological studies have demonstrated that humans are in fact extremely poor at discerning patterns and will frequently perceive patterns in random data and fail to perceive them in non-random data.[3] Gilovich et al. suggest that “people's intuitive conceptions of randomness depart systematically from the laws of chance” in two significant respects. Firstly, there is the gambler's fallacy, in which the perceived probability of an outcome is affected by observed previous outcomes; for example, an outcome of tails is seen as more likely when preceded by a long sequence of heads.[4] In basketball, for example, there is a widespread belief among both fans and players that the chance of a player scoring is higher when the shot is preceded by a series of successful shots. Despite this, a study by Gilovich et al. failed to provide substantive statistical evidence in support of the belief in shooting streaks, which seem instead to reflect the gambler's fallacy.[5]

Secondly, studies reveal that people tend to expect the law of large numbers to apply to small samples as well as to large ones.[6] This notion may lead people to reject the randomness of random sequences if the latter are seen as nonrepresentative of the laws of chance.The law of large numbers states that the observed average of results obtained from a series of trials will tend towards the expected relative frequency as the number of trials increases. The expected relative frequency of heads in a single fair coin toss, for example, is calculated as 0.5 (number of outcomes in event/number of possible outcomes). The observed relative frequency of heads will approach 0.5 as the number of trials increases, but only after a sufficiently large number of trials have been performed:

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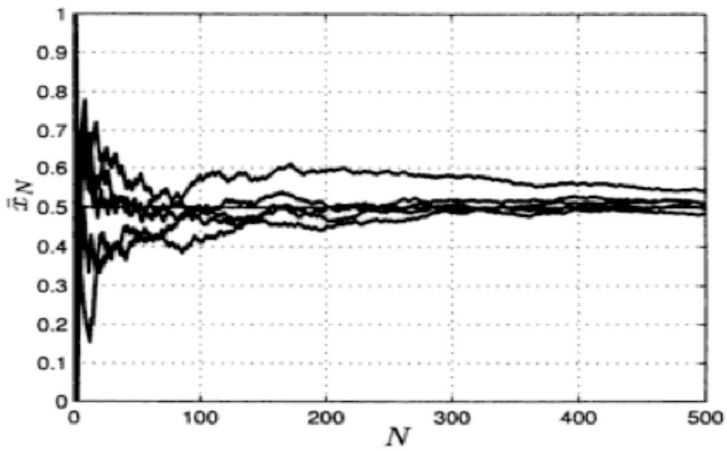
3 Daniel Kahneman and Amos Tversky, 'Subjective Probability: A Judgment of Representativeness', *Cognitive Psychology* 3 (1972): 430-435.
4 Ibid.
5 Ibid., 312.
6 Thomas Gilovich, Robert Vallone and Amos Tversky, 'The Hot Hand in Basketball: On the Misperception of Random Sequences', *Cognitive Psychology* 17 (1985): 296.



The Law of Large Numbers applied to coin tosses.[7]

Human subjects tend to assume that the observed relative frequency of a random event should be close to the expected relative frequency even in short sequences, and to perceive events as non-random where this expectation is not met. A sequence of coin tosses is therefore less likely to be seen as random if it includes a run of several heads in a row, although this outcome may be quite likely when taking into account the length of the entire sequence.[8] This tendency suggests that intuitive concepts of randomness are heavily influenced by subjective perceptions of meaning, since a run of heads or a run of an apparently non-random pattern (ABAB, for example) is more likely to be seen as significant and improbable and therefore non-random than a sequence in which no pattern can be detected.

Events that are scientifically proven to be non-random may be experienced as random by less informed subjects, whereas events considered random by scientists may be described as non-random by observers. The limitations and inaccuracy of our intuitions about randomness suggests the difficulty of defining randomness in terms of our subjective experience of unpredictability. In addition to these limitations, which may be considered to some extent as intrinsic tendencies in humans, a further epistemological difficulty can be identified. In observing apparently random phenomena, it is impossible to know whether an event is unpredictable because it is ontologically random, or whether this unpredictability is the result of insufficient knowledge of the conditions and processes that produce the event. It is therefore practically impossible to state with any certainty whether an event is 'genuinely' or ontologically random, or whether it is only apparently or 'epistemically' random. In some instances, epistemic differences between individuals may mean that an event is random to one individual, and non-random to another. Generally, the removal of these epistemic differences will result in the resolution of such disagreements. For example, a person

7 Steven M. Kay, *Intuitive Probability And Random Processes Using Matlab* (New York: Springer, 2006), 490.
8 Gilovich et al., 'The Hot Hand in Basketball', 296.

observing a non-random sequence produced by an algorithm will no longer perceive it as random if the mathematical processes that generate it are explained.

In other instances, however, it may be more difficult to define whether an event or process is random or not. If I wanted to study the distribution of a particular trait in a population – say the number of redheads in Melbourne – one way in which I could acquire a random sample of the population would be to select for another, unrelated trait – for example, being born at 2:39 P.M. To the subjects chosen for the study, the process of selection would appear entirely random and unpredictable. To me, however, possessing full knowledge of the process of selection, it would be entirely predictable. Does this mean that this process is genuinely random, or not? Antony Eagle rejects any split between "randomness by design" and naturally occurring randomness. The products of randomness by design behave in precisely the same way, Eagle argues, as random sequences, and are therefore genuinely random.[9]

Eagle's position is a consequence of his attempt to produce a definition of randomness that incorporates all scientific uses of the term. Yet since Eagle defines randomness as unpredictability, and the process of selection is clearly predictable to the researcher, the argument that such a process is genuinely random appears problematic. The difficulty can be resolved if we are willing to concede that there may be valid reasons for differentiating between kinds of randomness. Random selection may be considered random because the selected sample is random *relative* to the object of study. This kind of randomness is an extrinsic property that emerges as a consequence of the interactions between the sample and the study question; it is defined, not by unpredictability, but by the assumed unrelatedness between two factors. A non-random process can generate an outcome that is random for the purposes of the study; it can also appear random to the subjects of the study. This latter point provides further evidence for the limitations of subjective experience in accurately distinguishing random from non-random processes and events. If non-random processes are frequently experienced as random by subjects, the extent to which intersubjective definitions of randomness might succeed in overcoming this difficulty seems worthy of discussion.

3. Intersubjective Randomness: Practical Virtues and Theoretical Gaps

We have noted some of the epistemic problems inherent in subjective notions of randomness. These stem firstly from the apparently poor human capacities for pattern detection, and secondly from differing perceptions of randomness between individuals as a result of unequal states

9 Antony Eagle, 'Randomness is Unpredictability', *Journal for the Philosophy of Science* 56 (2005): 784.

of knowledge. This also raises the question of whether an event can be legitimately classified as random if there are disagreements about randomness between individuals. The perception of randomness is therefore closely linked to the observer's knowledge about the processes or initial conditions preceding an outcome. Reflecting on these issues in subjective interpretations of randomness, the question arises as to whether *all* randomness might be purely epistemic: the consequence of our inability to apprehend the relevant information about, and discern patterns in, events preceding an apparently random outcome. The extent to which Antony Eagle's definition of randomness, which may be classified as an intersubjective definition of randomness, provides a solution to the epistemic problems engendered in the subjective account, will now be discussed.

Eagle is not concerned with providing a definition of randomness that attributes strict ontic properties to randomness. Rather, Eagle's theory is motivated by the desire to provide a practical, working definition which encompasses four competing requirements in the scientific application of concepts of randomness. Firstly, a concept of randomness must be able to both produce and identify random sequences. Secondly, it must apply to unique events and finite phenomena, not merely to sequences or mass phenomena. Thirdly, a concept of randomness must be able to explain why certain systems exhibit random behaviours, and must be amenable to empirical confirmation. Finally, a concept of randomness must be compatible with determinism, "else we cannot explain the use of randomness to describe processes in population genetics or chaotic dynamics."[10]

Eagle's definition of randomness as unpredictability is qualified by the requirement of an intersubjective consensus on randomness within a relevant epistemic community.[11] Intersubjectivity in this context refers to the agreement of epistemic agents of a particular group on a given principle or concept. According to Eagle, since the parameters that determine predictability within a scientific theory "do not vary freely and without constraint from agent to agent but are subject to norms fixed by the communities of which agents are a part", then "rational agents cannot easily disagree over randomness, and . . . purely personal and subjective features of those agents do not play a significant role in judgements of randomness."[12] The requirement for an intersubjective consensus within an epistemic community thus allows for a definition of randomness as unpredictability that is not obviously subjective or relativistic.

In his proposal to consider randomness as unpredictability, Eagle provides the following definition for unpredictability:

10 Ibid., 756.
11 Ibid., 785.
12 Ibid., 783.

> An event E (at some temporal distance t) is unpredictable for a predictor P iff P's posterior credence in E after conditioning on current evidence and the best prediction function available to P is not 1, that is, if the prediction function yields a posterior probability distribution that does not assign probability 1 to E.[13]

The posterior probability of an event in Bayesian probability theory is given by revising a "prior" probability of an event on the basis of newly acquired information. This process of revision is known as conditioning.[14] On Eagle's definition then, any event is unpredictable if a predictor, following conditioning on available evidence, fails to predict it to 100% probability; or, an event E is unpredictable for a predictor P if P failed to predict it, given that P had taken into account all the relevant evidence and information. Eagle goes on to define a random event as one that is maximally unpredictable:

> An event E is random for a predictor P using theory T iff E is maximally unpredictable. An event E is maximally unpredictable for P and T iff the posterior probability of E yielded by the prediction functions that T makes available, conditional on current evidence, is equal to the prior probability of E.[15]

In other words, Eagle considers an event to be maximally unpredictable if the probability of an event is the same whether or not the current evidence is taken into account. Frequently, a phenomenon may be maximally unpredictable if considered as a unique event, but relatively predictable when considered as a process. For example, probabilistic theories can accurately predict the process of radioactive decay, but cannot predict when a particular atom will decay. Radioactive decay is therefore maximally unpredictable (and therefore random) with regard to single events, since the theory that attempts to predict when a particular atom will decay cannot do so with any more accuracy than a theory based on probabilistic chance.

Another way of conceptualising Eagle's notion of randomness as unpredictability is to consider the apparent independence of random outcomes from their past and present observed states.[16] Radioactive decay is random because no apparent information about an atom enables an

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13 Ibid., 772.
14 James Joyce, 'Bayes' Theorem', *The Stanford Encyclopedia of Philosophy (Fall 2008 Edition)*, ed. Edward N. Zalta, <http://plato.stanford.edu/archives/fall2008/entries/bayes-theorem/>. Accessed 18.06.2012.
15 Eagle, 'Randomness is Unpredictability', 775.
16 Ibid., 776.

observer to provide better-than-chance predictions about when that atom will decay. Similarly, a coin toss is random because the information available in the initial state of the coin – its weight, its size, the metals that it is composed of – appears to be entirely independent of the outcome heads or tails. However, the latter example raises an interesting question for Eagle's account, since Persi Diaconis's work has shown that coin tosses are not random when flipped under controlled conditions.[17] Yet they still *appear* random to individuals who flip them and place bets on the outcome.[18] Coin tosses by most humans are still highly unpredictable, but it seems possible given Diaconis's findings that this is not because of any inherent property of the process of coin-tossing, but rather because the computational complexity entailed by human involvement is too great to accurately predict the outcome of the coin toss. Furthermore, improvements in computational capacities have enabled statisticians to perceive non-random patterns in pseudo-random sequences generated by numerical methods.[19] The results of the Diaconic experiments demonstrate that even within epistemic communities, concepts of randomness may be fluid and subject to revision.

What does this entail for Eagle's definition of randomness as unpredictability? The intersubjective requirement in Eagle's theory, while imposing limitations on what may be considered random, also provides sufficient conceptual flexibility to incorporate changes in perceptions of randomness such as those engendered by Diaconis's research on coin-tossing. On Eagle's account, randomness (i.e. maximal unpredictability) is defined with reference to a predictor that is (presumably) considered by the scientific community to constitute the best predictor for a given event or process.[20] As Eagle observes, the parameters for experiment and measurement are fixed by the scientific community, yielding a definition of randomness that is not arbitrarily subjective; yet this does not preclude that these parameters themselves might be subject to re-evaluation and modification in light of new empirical evidence.[21] The discovery of the non-random nature of a process previously believed within the scientific community to be random does not present a significant challenge to Eagle's hypothesis. However, it does suggest that randomness on this account possesses a certain plasticity in relation to evolving paradigms within epistemic communities.

Eagle's definition of randomness as maximal unpredictability, predicated on an intersubjective consensus, responds effectively to the four key requirements for scientific

17 Persi Diaconis et al., 'Dynamical Bias in the Coin Toss', *SIAM Review* 49 no. 2 (April 2007): 211-235.

18 Interestingly, however, there is a slight bias for the coin to land facing the same way as when it started. See Diaconis et al., 'Dynamical Bias in the Coin Toss'.

19 H. Inoue et al., 'Random Numbers Generated by a Physical Device', *Journal for the Royal Statistical Society* 32 no. 2 (1983): 115.

20 Eagle, 'Randomness is Unpredictability', 775.

21 Ibid., 783.

application outlined in the beginning of his article. It allows for randomness in statistical testing as sequences can be produced that are unpredictable to test subjects; it enables single events to be random, since they can be maximally unpredictable; it is amenable to empirical testing and refutation, since predictions that perform better than chance are a likely indicator that a process is not random; finally, it is compatible with determinism, since unpredictability can occur as a result of many reasons that are independent of indeterminism.[22]

For practical and scientific purposes, therefore, Eagle's definition of randomness is extremely useful and comprehensive. Furthermore, Eagle's account goes some way towards resolving the kinds of epistemic difficulties we have observed in subjective concepts of randomness. It does so firstly by distinguishing randomness from something like “personal unpredictability”, which can be shown to be merely a reflection of the “ignorance and incapability” of an individual.[23] True randomness must be that which is defined by unpredictability relative to theories that have achieved consensus within the scientific community. This definition provides a concept of randomness that is not arbitrary, but objective relative to agreed standards within the relevant epistemic community.[24] Randomness cannot fluctuate according to subjective experience, but may be redefined if these standards or theories are modified as a result of new empirical evidence, as with the Diaconis experiments. Eagle's intersubjective definition, therefore, sets strong but not absolute limits on concepts of randomness by requiring that these conform to scientific standards.

Although Eagle's definition avoids some of the epistemic problems engendered by subjective conceptions of randomness, it does not completely remove them. Eagle's definition does not attempt to determine whether randomness constitutes an intrinsic feature of reality or is merely the product of epistemic limitations. Eagle is concerned with randomness “in practice” - that is, observed randomness – rather than randomness “in principle”, or ontological randomness.[25] For Eagle, therefore, whether unpredictability is a result of an indeterministic system, or the result of epistemic constraints in analysing a deterministic system, is irrelevant to classifying a system as random or not.[26] While this definition is extremely useful in its practical applications, the question of whether or not randomness might constitute an ontic feature of reality lies outside Eagle's purpose. Yet the question of whether randomness might have an ontological explanation, and what

22 Ibid., 786.
23 Ibid., 779.
24 Ibid.
25 Soubhik Chakraborty, 'On “Why” and “What” of Randomness', *DataCrítica: International Journal of Critical Statistics*, 3 no. 2, (2010): 2.
26 Eagle, 'Randomness as Unpredictability', 786-87.

such an explanation would entail, is nevertheless an important one, since it will have implications for our understanding of the nature of physical reality.

3. Properties of Randomness in Mathematical Sequences: Von Mises's Account of Randomness

We have suggested that the problem of determining whether randomness is an intrinsic feature of reality or merely a product of epistemic limitations is the most significant obstacle to developing a theory of randomness. Recent experiments have shown that processes previously thought to be random are in fact non-random; on this evidence, it seems likely that an increasing number of processes currently considered random will be able to be explained and predicted as scientific knowledge evolves and measuring techniques become more sophisticated. Even if some events can never be entirely predictable, this does not imply that they are ontically random; they might simply be too sensitive to the disturbances involved in taking measurements for their premeasurement state to be determined with sufficient accuracy to make reliable predictions.[27] This kind of evidence seems to suggest that randomness is more likely a feature of epistemic limitations than of ontic properties. If we define randomness as unpredictability and assume that all randomness is epistemic, this entails a deterministic view of reality.[28] Conversely, an ontic conception of randomness implies indeterminism.[29] The ontic status of randomness is therefore directly connected to conceptions of physical reality.

Empirical approaches are unlikely to permit us to postulate as to whether randomness might have ontic properties, since from an operational perspective ontic randomness might be indistinguishable from pseudorandom but highly complex phenomena.[30] A better approach might be to consider the kinds of properties randomness would be required to possess in order to produce the empirical effects observed in random processes and events. The definitions of randomness proposed by Richard von Mises differ from Eagle's theory in that they attempt to define sequences as random by virtue of the properties of those sequences, rather than with reference to epistemic qualities such as unpredictability. The properties attributed to random sequences in von Mises's account might be analysed in order to speculate on the kinds of processes that would be necessary in order to produce them, and whether these might provide a model for ontic randomness.

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27 Ibid., 766.
28 Irving John Good, 'Random Thoughts about Randomness', *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* (1972): 122.
29 Chakraborty, 'On "Why" and "What" of Randomness', 1.
30 Good, 'Random Thoughts about Randomness', 127.

Von Mises's account of randomness represents the first "detailed and sophisticated attempt" to provide a definition of randomness for a sequence with a single stochastic property.[31] Von Mises's theory is only intended to apply to "mass phenomena and repetitive events"; that is, to *infinite* sequences or series of phenomena, and not to unique events.[32] A mathematical sequence, or a sequence of events, is defined as random on von Mises's account if none of its constituent elements yields any predictive information about any other element in that sequence. For example, if a subsequence of a sequence $x_1, \dots, x_{n-1}$, is truly random, then neither the values of previous elements in the sequence, nor the place of the desired value in the sequence, will be of any use in predicting the value of x_n.*33* In order to demonstrate this, von Mises defines a method of producing subsequences from the original sequence that he labels 'admissible place selection'. Although von Mises has been criticised for his failure to adequately formalise his definition, an 'admissible place selection' can be generally understood as "place selection is a procedure for selecting a subsequence of the given sequence x in such a way that the decision to select a term X_n does not depend on the value of X_n."34 On von Mises' definition, the instruction: Pick every X_n where *n* is a prime number is an admissible place selection, as it does not take into account the value of X_n when choosing X_n.35 If a sequence is genuinely random, the subsequence produced by an admissible place selection will possess the same limiting digit frequencies as the original sequence. In an infinite random sequence with two possible values for X_n, 1 and 0, any subsequence produced by admissible place selection will meet the property of large numbers; both values will have a limiting relative frequency of 0.5.36 A random sequence for von Mises is one in which it is impossible to produce a biased subsequence.

A random sequence for von Mises, therefore, is one in which there is no exploitable regularity; nothing in an element or sequence of elements gives any information about any other part of or element in the sequence.37 The manifestation of randomness as it emerges on von Mises's account might be defined as *the mutual independence of all constituent elements in a sequence such that where x represents a given value for an element and y represents the total number of element values in a sequence, the probability of x for any place in the sequence is equal to x/y.* This kind of

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31 Antony Eagle, 'Chance versus Randomness', *The Stanford Encyclopedia of Philosophy (Spring 2012 Edition)*, ed. Edward N. Zalta, <http://plato.stanford.edu/archives/spr2012/entries/chance-randomness/>. Accessed 25.05.2012.

32 Richard von Mises, 'On the Foundations of Probability and Statistics', *The Annals of Mathematical Statistics* 12, no 1 (June, 1941): 191.

33 Eagle, 'Chance Versus Randomness'.

34 Michiel van Lambalgen, 'Von Mises' Definition of Random Sequences Reconsidered', *Journal of Symbolic Logic* 52 no. 3 (Sep., 1987): 725; Richard von Mises, *Probability, Statistics and Truth* (New York: Dover, 1957), 25.

35 Van Lambalgen, 'Von Mises' Definition of Random Sequences Reconsidered', 727.

36 Eagle, 'Chance Versus Randomness'.

37 Ibid.

notion is a *product notion of randomness*; it defines random sequences in terms of their properties, but does not address the randomness of the processes that produce them.38 However, von Mises randomness has been shown to be too inclusive. In the 'random walk' model of random (binary) sequences, each element in a sequence is interpreted as a step to the left for 0s, and a step to the right for 1s. In certain sequences which are random on von Mises' definition, a bias in the initial segment of the sequence results in a 'walk' that always stays to the right.39 Sequences that approach their limiting frequency from one side could be exploited by a simple betting strategy, and are therefore not genuinely random.40

4. Properties of Process Randomness: A Model for Ontic Randomness?

If properties of product randomness could be deduced from von Mises's account of process randomness, this would provide evidence for randomness as an operative law in empirical reality, rather than a potential reflection of epistemic limitations. However, this kind of deduction is extremely difficult to make when considering the kinds of processes underlying the product randomness that von Mises is addressing. Von Mises is generally concerned with randomness as it relates to gambling systems. However, if we consider Diaconis's experiments, it seems improbable that something like a roulette wheel will be a genuinely random process; the randomness of roulette numbers is more likely to be a product of incalculability rather than intrinsic unpredictability. Furthermore, von Mises' definition is intended to apply only to infinite sequences, making its application to practical instances problematic.

However, Andrey Kolmogorov proposed a definition of randomness that could apply to finite sequences. Like von Mises, Kolmogorov perceived that a property of random sequences was a lack of regularity or predictability. However, rather than defining randomness in terms of the characteristics of subsequences produced by admissible place selection, Kolmogorov argued that random sequences could be best defined by their incompressibility.41 A non-random sequence can be produced by an algorithm whose length is significantly shorter than its own. For example, the sequence:

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38 Ibid.
39 Ibid.
40 Ibid; Persi Diaconis, 'Computation and Randomness', 10 Great Ideas about Chance, Stanford University, Spring 2012. Downloaded from: http://www-stat.stanford.edu/~cgates/PERSI/Courses/Phil166-266/. Accessed 18.06.2012.
41 Eagle, 'Chance and Randomness'.

is not random on Kolmogorov's definition, as it is the decimal expansion of $\sqrt{2} - 1$.[42] A random sequence on Kolmogorov's definition will be one such that the shortest algorithm that can produce it will be approximately the same length as the sequence itself.[43]

As Kolmogorov's definition applies to finite sequences, it might be more amenable to application to observable phenomena. Like von Mises' definition, Kolmogorov's is a product definition of randomness, but might provide insight into the kinds of processes that could produce random effects. Imagine, for example, an experiment in which a quantity of radioactive atoms was assembled such that the probability of one atom in the group decaying after ten minutes was equal to 0.5. On average, therefore, we would expect three atoms to decay every hour. Imagine also that it is possible to assign a numerical value to every atom in the experiment, so that each decayed atom can be identified by its pre-assigned value. By recording the value of each atom as it decayed, it would be possible to produce a numerical sequence that would conform to Kolmogorov's conditions of randomness.

The use of radioactive decay as a means of generating random sequences is well established in statistical practices and has been demonstrated to produce more reliably random sequences than ordinary numerical methods.[44] Particle radiation might therefore provide an example of a genuinely random *process*, which might be analysed in order to consider the kinds of ontic properties randomness would have to possess in order for it to produce the effects observed in the sequences they produce. In the case of the roulette wheel, it is relatively easy to conceive of the kinds of factors – angle and speed of the throw, rotating speed of the wheel, position of the wheel at the point of contact, et cetera – that, if analysed, could be used to accurately predict the outcome. But what kinds of factors might determine whether an particle decays at a particular moment? Either we must postulate that particle radiation is also regulated by deterministic processes; or we must concede that the randomness of particle radiation reflects the ontically indeterministic nature of reality.[45]

Particle radiation represents an apparently random process that could result in the properties of product randomness as defined by Kolmogorov. As remarked, it is entirely possible that particle radiation is a non-random process whose operations have yet to be fully discovered. Within a deterministic framework, two main possibilities may be evoked to explain our experience

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42 Thomas M. Cover and Joy A. Thomas, *Elements of Information Theory* (New York: Wiley Interscience, 1991), 145.
43 Ibid.
44 Inoue et. al., 'Random Numbers Generated by a Physical Device', 120.
45 Chakraborty, 'On "Why" and "What" of Randomness', 1.

of randomness in observable phenomena. Bohmian mechanics is a deterministic version of quantum theory that states that quantum randomness arises only as a result of our ignorance of initial conditions.[46] According to this theory, our ignorance of certain, nonlocal "hidden variables" in quantum mechanical processes prevents us from making accurate predictions about individual outcomes. Furthermore, since Bohmian mechanics recovers the Heisenberg uncertainty relations (which states that certain pairs physical properties of particles cannot be simultaneously known, such as position and momentum), we can "never be in a position to know enough hidden variables to violate the appearance of random outcomes."[47] The Bohmian theory of quantum mechanics presents randomness as purely, but insurmountably, epistemic. Bohmian mechanics has witnessed a revival in popularity in recent years, and may provide a feasible explanation for the persistence of random phenomena in deterministic terms.[48]

A second possibility for an explanation of randomness within a deterministic view is provided by the Many Worlds Interpretation (MWI) of quantum mechanics. The MWI asserts that in addition to the world of which we are directly aware, there exist many other similar words parallel in space and time.[49] According to this version of quantum mechanics, where there are two or more possible outcomes for an event, both or all occur, resulting in a splitting of worlds. The Schrödinger's cat thought experiment provides a useful means of examining the implications of the MWI with reference to radioactive decay, which we have highlighted above as a likely candidate for a genuinely random process. A cat is sealed inside a box containing a cylinder filled with hydrogen cyanide and a radioactive substance. The decay of a single atom will trigger the release of the cyanide, thus killing the cat. The probability of this occurring after one hour is approximately 0.5. According to the MWI, the world in which the atom decays (and the cat is dead), *and* the world in which it does not (and the cat is alive), both exist.[50] Randomness, on this account, does not exist, as all possible outcomes of an event occur; it is merely an illusion produced by our inability to experience the worlds in which the alternative outcomes exist.[51]

If, however, particle radiation represented a genuinely random and inherently

46 Geoffrey Hellman, 'Interpretations of Probability in Quantum Mechanics: A Case of "Experimental Metaphysics"' in *Quantum Reality, Relativistic Causality, and Closing the Epistemic Circle: Essays in Honour of Abner Shimony* ed. Wayne C. Myrvold and Joy Christian (Springer, 2009), 225.

47 Ibid.

48 Stephen L. Adler, 'Probability in Orthodox Quantum Mechanics: Probability as a Postulate Versus Probability as an Emergent Phenomenon' in *Chance in Physics: Foundations and Perspectives* ed. J Bricmont et al. (Berlin: Springer, 2001), 103.

49 Lev Vaidman, 'Many-Worlds Interpretation of Quantum Mechanics', *The Stanford Encyclopedia of Philosophy (Fall 2008 Edition)*, ed. Edward N. Zalta, <http://plato.stanford.edu/archives/fall2008/entries/qm-manyworlds/>. Accessed 18.06.2012.

50 Ibid.

51 Alan A. Grometstein, *The Roots of Things: Topics in Quantum Mechanics* (New York: Kluwer, 1999), 10.

unpredictable process, it would be necessary to give an account of reality that could incorporate randomness as an ontological property. An alternative explanation for our experience of randomness in quantum events might therefore lie with an indeterministic interpretation of quantum mechanics. In an ontic indeterminist framework, random events are those which are intrinsically unpredictable as there are no factors that fully determine their outcome.[52] Collapse theories of quantum mechanics raise the possibility for ontic randomness. Such theories posit that the deterministic Schrödinger wavefunction collapses at certain points in time – most noticeably under measurement and observation – and that the process of collapse is indeterministic.[53] However, such theories have been criticised on the grounds that the collapse process itself is vague and physically ill-defined.[54] Nevertheless, the fact that there are "both deterministic and indeterministic versions of quantum mechanics that are empirically equivalent to the best of our current knowledge"[55] suggests that ontic randomness remains a possible explanation for our empirical experiences of randomness.

Conclusion

Given the empirical difficulties, it seems unlikely that we will ever conclusively determine whether randomness constitutes an intrinsic feature of reality, or whether what appears as randomness is merely the result of our own computational and epistemic limitations. The discovery that some processes previously considered random are produced by non-random processes suggests the possibility that randomness is merely an epistemic phenomenon, and that the number of processes considered to be random will decrease proportionate to increases in scientific knowledge and technological capabilities. The possibility of an explanation for observed randomness in deterministic versions of quantum mechanics seems promising in view of the apparent increase in the number of processes revealed to be deterministic. Yet the history of science also contains examples of processes previously thought to be non-random and deterministic that are now widely considered to be indeterministic. The works of John Earman, for example, have shown that there are examples of indeterministic processes in classical mechanics, which has been traditionally considered as deterministic.[56] Yet as science stands today, empirical observation provides examples of apparently random and demonstrably non-random processes, deterministic and indeterministic systems; for the moment, there is no way to ascertain whether this state of knowledge reflects the intrinsic properties of reality, or merely our ignorance of it.

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52 Robert Kane, *The Oxford Handbook of Free Will* (Oxford: Oxford University Press, 2011), 90.
53 Hoefer, 'Causal Indeterminism'.
54 Ibid.
55 Kane, *The Oxford Handbook of Free Will,* 93.
56 John Earman, 'Determinism: What We Have Learned and What We Still Don't Know' in J. K. Campbell et al (eds.), *Determinism, Freedom, and Agency*, 21-46 (Cambridge, MA: MIT Press, 2004).

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