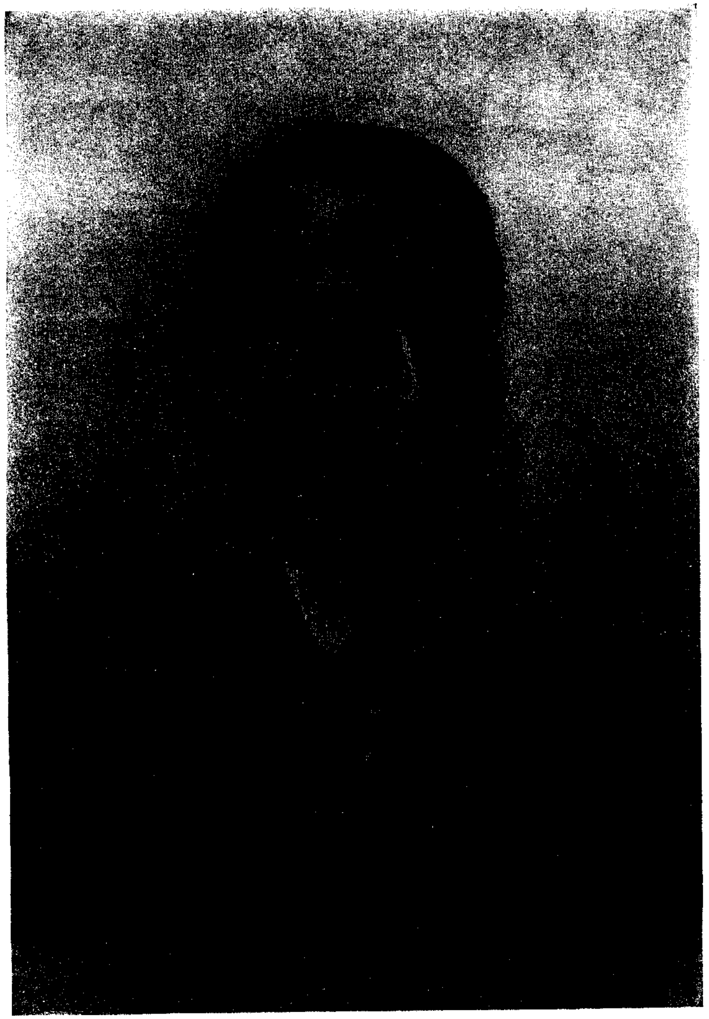

PROFESSOR ERNST MACH

1838 - 1916

THE

SCIENCE OF MECHANICS

A CRITICAL AND HISTORICAL ACCOUNT
OF ITS DEVELOPMENT

BY

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TRANSLATED FROM THE GERMAN

BY

THOMAS J. McCORMACK

CONTAINING ADDITIONS AND ALTERATIONS
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PREFACE TO THE FIFTH ENGLISH EDITION

The former editions of the English translation of Mach's *Mechanics* with appendix and supplementary volume seemed so unwieldy and difficult to peruse that, in preparing a new edition, we have felt it imperative to incorporate the alterations and additions into the text, which has been revised according to the ninth German edition. With a few minor corrections, the text remains substantially as translated by Mr. McCormack.

In the seventh German edition Professor Mach made extensive changes; considerable material was added and some omitted. Mr. P. E. B. Jordain in 1911 in consultation with the author signified these alterations in a supplementary volume, all of which except Mr. Jordain's own notes has been included in the present edition. These notes are still available in the supplementary volume. The most extensive additions relate to historical researches on the work of Galileo's precursors and the early work of Galileo himself. The book is dedicated to Ernst Wohlwill of whose researches much use has been made.

A translation of Section 8, Chapter III has been added in place of the statement of Mr. C. S. Peirce trary coördinates. The difference between the analytic and the synthetic method is again done away with, and the advantages of the two are combined. The kindred procedure of Hamilton, which has been illustrated by an example on page 196, will give some idea of these advantages.

IV.

THE ECONOMY OF SCIENCE.*

1. It is the object of science to replace, or *save*, experiences, by the reproduction and anticipation of facts in thought. Memory is handier than experience, and often answers the same purpose. This economical office of science, which fills its whole life, is apparent at first glance; and with its full recognition all mysticism in science disappears. Science is communicated by instruction, in order that one man may profit by the experience of another and be spared the trouble of accumulating it for himself; and thus, to spare posterity, the experiences of whole generations are stored up in libraries.

Language, the instrument of this communication, is itself an economical contrivance. Experiences are analyzed, or broken up, into simpler and more familiar experiences, and then symbolized at some sacrifice of precision. The symbols of speech are as yet restricted in their use within national boundaries, and doubtless will long remain so. But written language is gradually

* *Cf.* my paper, "Die Leitgedanken meiner naturwissenschaftlichen Erkenntnislehre und ihre Aufnahme durch die Zeitgenossen" (*Scientia: Rivista di Scienza*, vol. vii, 1910, No. 14, 2; or *Physikalische Zeitschrift*, 1910, pp. 599-606).

being metamorphosed into an ideal universal character. It is certainly no longer a mere transcript of speech. Numerals, algebraic signs, chemical symbols, musical notes, phonetic alphabets, may be regarded as parts already formed of this universal character of the future; they are, to some extent, decidedly conceptual, and of almost general international use. The analysis of colors, physical and physiological, is already far enough advanced to render an international system of color-signs perfectly practical. In Chinese writing, we have an actual example of a true ideographic language, pronounced diversely in different provinces, yet everywhere carrying the same meaning. Were the system and its signs only of a simpler character, the use of Chinese writing might become universal. The dropping of unmeaning and needless accidents of grammar, as English mostly drops them, would be quite requisite to the adoption of such a system. But universality would not be the sole merit of such a character; since to read it would be to understand it. Our children often read what they do not understand; but that which a Chinaman cannot understand, he is precluded from reading.

2. In the reproduction of facts in thought, we never reproduce the facts in full, but only that side of them which is important to us, moved to this directly or indirectly by a practical interest. Our reproductions are invariably abstractions. Here again is an economical tendency.

Nature is composed of sensations as its elements. Primitive man, however, first picks out certain compounds of these elements—those namely that are relatively permanent and of greater importance to him. The first and oldest words are names of "things."

Even here, there is an abstractive process, an abstraction from the surroundings of the things, and from the continual small changes which these compound sensations undergo, which being practically unimportant are not noticed. No inalterable thing exists. The thing is an abstraction, the name a symbol, for a compound of elements from whose changes we abstract. The reason we assign a single word to a whole compound is that we need to suggest all the constituent sensations at once. When, later, we come to remark the changeableness, we cannot at the same time hold fast to the idea of the thing's permanence, unless we have recourse to the conception of a thing-in-itself, or other such like absurdity. Sensations are not signs of things; but, on the contrary, a thing is a thought-symbol for a compound sensation of relative fixedness. Properly speaking the world is not composed of "things" as its elements, but of colors, tones, pressures, spaces, times, in short what we ordinarily call individual sensations.

The whole operation is a mere affair of economy. In the reproduction of facts, we begin with the more durable and familiar compounds, and supplement these later with the unusual by way of corrections. Thus, we speak of a perforated cylinder, of a cube with beveled edges, expressions involving contradictions, unless we accept the view here taken. All judgments are such amplifications and corrections of ideas already admitted.

3. In speaking of cause and effect we arbitrarily give relief to those elements to whose connection we have to attend in the reproduction of a fact in the respect in which it is important to us. There is no cause nor effect in nature; nature has but an individual existence; nature simply *is*. Recurrences of like cases in

which A is always connected with B, that is, like results under like circumstances, that is again, the essence of the connection of cause and effect, exist but in the abstraction which we perform for the purpose of mentally reproducing the facts. Let a fact become familiar, and we no longer require this putting into relief of its connecting marks, our attention is no longer attracted to the new and surprising, and we cease to speak of cause and effect. Heat is said to be the cause of the tension of steam; but when the phenomenon becomes familiar we think of the steam at once with the tension proper to its temperature. Acid is said to be the cause of the reddening of tincture of litmus; but later we think of the reddening as a property of the acid.

Hume first propounded the question: How can a thing A act on another thing B? Hume, in fact, rejects causality and recognizes only a wonted succession in time. Kant correctly remarked that a *necessary* connection between A and B could not be disclosed by simple observation. He assumes an innate idea or category of the mind, a *Verstandesbegriff*, under which the cases of experience are subsumed. Schopenhauer, who adopts substantially the same position, distinguishes four forms of the "principle of sufficient reason" — the logical, physical, and mathematical form, and the law of motivation. But these forms differ only as regards the matter to which they are applied, which may belong either to outward or inward experience.

The natural and common-sense explanation is apparently this. The ideas of cause and effect originally sprang from an endeavor to reproduce facts in thought. At first, the connection of A and B, of C and D, of E and F, and so forth, is regarded as familiar. But after a greater range of experience is acquired and a connection between M and N is observed, it often turns out that we recognize M as *made up of* A, C, E, and N of B, D, F; the connection of which was before a *familiar* fact and accordingly possesses with us a higher authority. This explains why a person of experience regards a new event with different eyes than the novice. The new experience is illuminated by the mass of old experience. As a fact, then, there really does exist in the mind an "idea" under which fresh experiences are subsumed; but that idea has itself been developed from experience. The notion of the *necessity* of the causal connection is probably created by our voluntary movements in the world and by the changes which these indirectly produce, as Hume supposed but Schopenhauer contested. Much of the authority of the ideas of cause and effect is due to the fact that they are developed *instinctively* and involuntarily, and that we are distinctly sensible of having personally contributed nothing to their formation. We may, indeed, say, that our sense of causality is not acquired by the individual, but has been perfected in the development of the race. Cause and effect, therefore, are things of thought, having an economical office. It cannot be said *why* they arise. For it is precisely by the abstraction of uniformities that we know the question "why."*

* In the text I have employed the term "cause" in the sense in which it is ordinarily used. I may add that with Dr. Carus, following the practice of the German philosophers, I ***distinguish*** "cause," or ***Realgrund***, from ***Erkenntnissgrund***. I also agree with Dr. Carus in the statement that "the signification of cause and effect is to a great extent arbitrary and depends much upon the proper tact of the observer." (See his ***Fundamental Problems***, pp. 79-91, Chicago: The Open Court Publishing Co., 1891. Also, p. 84.)

The notion of cause possesses significance only as a means of provisional knowledge or orientation. In any exact and profound investigation of an event the inquirer must regard the phenomena as ***dependent*** on ***one another*** in the same way that the geometer regards the sides and angles of a triangle as dependent on one another. He will constantly keep before his mind, in this way, all the conditions of fact.

4. In the details of science, its economical character is still more apparent. The so-called descriptive sciences must chiefly remain content with reconstructing individual facts. Where it is possible, the common features of many facts are once for all placed in relief. But in sciences that are more highly developed, rules for the reconstruction of great numbers of facts may be embodied in a *single* expression. Thus, instead of noting individual cases of light-refraction, we can mentally reconstruct all present and future cases, if we know that the incident ray, the refracted ray, and the perpendicular lie in the same plane and that $\sin \alpha / \sin \beta = n$. Here, instead of the numberless cases of refraction in different combinations of matter and under all different angles of incidence, we have simply to note the rule above stated and the values of n, which is much easier. The economical purpose is here unmistakable. In nature there is no *law* of refraction, only different cases of refraction. The law of refraction is a concise compendious rule, devised by us for the mental reconstruction of a fact, and only for its reconstruction in part, that is, on its geometrical side.

5. The sciences most highly developed economically are those whose facts are reducible to a few numerable elements of like nature. Such is the science of mechanics, in which we deal exclusively with spaces, times, and masses. The whole previously established economy of mathematics stands these sciences in stead. Mathematics may be defined as the economy of counting. Numbers are arrangement-signs which, for the sake of perspicuity and economy, are themselves arranged in a simple system. Numerical operations, it is found, are independent of the kind of objects operated on, and are consequently mastered once for all. When,

for the first time, I have occasion to add five objects to seven others, I count the whole collection through, at once; but when I afterwards discover that I can start counting from 5, I save myself part of the trouble; and still later, remembering that 5 and 7 always count up to 12, I dispense with the numeration entirely.

The object of all arithmetical operations is to *save* direct numeration, by utilizing the results of our old operations of counting. Our endeavor is, having done a sum once, to preserve the answer for future use. The first four rules of arithmetic well illustrate this view. Such, too, is the purpose of algebra, which, substituting relations for values, symbolizes and definitively fixes all numerical operations that follow the same rule. For example, we learn from the equation

$$\frac{x^2 - y^2}{x + y} = x - y,$$

that the more complicated numerical operation at the left may always be replaced by the simpler one at the right, whatever numbers x and y stand for. We thus save ourselves the labor of performing in future cases the more complicated operation. Mathematics is the method of replacing in the most comprehensive and *economical* manner possible, *new* numerical operations by old ones done already with known results. It may happen in this procedure that the results of operations are employed which were originally performed centuries ago.

Often operations involving intense mental effort may be replaced by the action of semi-mechanical routine, with great saving of time and avoidance of fatigue. For example, the theory of determinants

owes its origin to the remark, that it is not necessary to solve each time anew equations of the form

$$a_1 x + b_1 y + c_1 = 0$$
$$a_2 x + b_2 y + c_2 = 0,$$

from which result

$$x = -\frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} = -\frac{P}{N}$$
$$y = -\frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} = -\frac{Q}{N},$$

but that the solution may be effected by means of the coefficients, by writing down the coefficients according to a prescribed scheme and operating with them *mechanically*. Thus,

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 = N$$

and similarly

$$\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = P, \text{ and } \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = Q.$$

By means of mathematical operations a complete relaxation of the mind can occur. This happens where operations of counting hitherto performed are symbolized by mechanical operations with signs, and our brain energy, instead of being wasted on the repetition of old operations, is spared for more important tasks. The merchant pursues a like economy, when, instead of directly handling his bales of goods, he operates with bills of lading or assignments of them. The drudgery of computation may even be relegated to a machine. Several different types of calculating machines are actually in practical use. The earliest of these (of any complexity) was the difference-engine of

Babbage, who was familiar with the ideas here presented.

A numerical result is not always reached by the *actual* solution of the problem; it may also be reached indirectly. It is easy to ascertain, for example, that a curve whose quadrature for the abscissa x has the value x^m, gives an increment $m\,x^{m-1}\,d\,x$ of the quadrature for the increment $d\,x$ of the abscissa. But we then also know that $\int m\,x^{m-1}\,d\,x = x^m$; that is, we recognize the quantity x^m from the increment $m\,x^{m-1}\,d\,x$ as unmistakably as we recognize a fruit by its rind. Results of this kind, accidentally found by simple inversion, or by processes more or less analogous, are very extensively employed in mathematics.

That scientific work should be more useful the more it has been used, while mechanical work is expended in use, may seem strange to us. When a person who daily takes the same walk accidentally finds a shorter cut, and thereafter, remembering that it is shorter, always goes that way, he undoubtedly saves himself the difference of the work. But memory is really not work. It only places at our disposal energy within our present or future possession, which the circumstance of ignorance prevented us from availing ourselves of. This is precisely the case with the application of scientific ideas.

The mathematician who pursues his studies without clear views of this matter, must often have the uncomfortable feeling that his paper and pencil surpass him in intelligence. Mathematics, thus pursued as an object of instruction, is scarcely of more educational value than busying oneself with the Cabala. On the contrary, it induces a tendency toward mystery, which is pretty sure to bear its fruits.

6. The science of physics also furnishes examples of this economy of thought, altogether similar to those we have just examined. A brief reference here will suffice. The moment of inertia saves us the separate consideration of the individual particles of masses. By the force-function we dispense with the separate investigation of individual force-components. The simplicity of reasonings involving force functions springs from the fact that a great amount of mental work had to be performed before the discovery of the properties of the force-functions was possible. Gauss's dioptrics dispenses us from the separate consideration of the single refracting surfaces of a dioptrical system and substitutes for it the principal and nodal points. But a careful consideration of the single surfaces had to precede the discovery of the principal and nodal points. Gauss's dioptrics simply *saves* us the necessity of often repeating this consideration.

We must admit, therefore, that there is no result of science which in point of principle could not have been arrived at wholly without methods. But, as a matter of fact, within the short span of a human life and with man's limited powers of memory, any stock of knowledge worthy of the name is unattainable except by the *greatest* mental economy. Science itself, therefore, may be regarded as a minimal problem, consisting of the completest possible presentment of facts with the *least possible expenditure of thought.*

7. The function of science, as we take it, is to replace experience. Thus, on the one hand, science must remain in the province of experience, but, on the other, must hasten beyond it, constantly expecting confirmation, constantly expecting the reverse. Where neither confirmation nor refutation is possible, science

is not concerned. Science acts and acts only in the domain of *uncompleted* experience. Exemplars of such branches of science are the theories of elasticity and of the conduction of heat, both of which ascribe to the smallest particles of matter only such properties as observation supplies in the study of the larger portions. The comparison of theory and experience may be farther and farther extended, as our means of observation increase in refinement.

Experience alone, without the ideas that are associated with it, would forever remain strange to us. Those ideas that hold good throughout the widest domains of research and that supplement the greatest amount of experience, are the *most scientific*. The principle of continuity, the use of which everywhere pervades modern inquiry, simply prescribes a mode of conception which conduces in the highest degree to the economy of thought.

8. If a long elastic rod be fastened in a vise, the rod may be made to execute slow vibrations. These are directly observable, can be seen, touched, and graphically recorded. If the rod be shortened, the vibrations will increase in rapidity and cannot be directly seen; the rod will present to the sight a blurred image. This is a new phenomenon. But the sensation of touch is still like that of the previous case; we can still make the rod record its movements; and if we mentally retain the *conception* of vibrations, we can still anticipate the results of experiments. On further shortening the rod the sensation of touch is altered; the rod begins to sound; again a new phenomenon is presented. But the phenomena do not all change at once; only this or that phenomenon changes; consequently the accompanying notion of vibration, which

is not confined to any single one, is still serviceable, still economical. Even when the sound has reached so high a pitch and the vibrations have become so small that the previous means of observation are not of avail, we still *advantageously* imagine the sounding rod to perform vibrations, and can predict the vibrations of the dark lines in the spectrum of the polarized light of a rod of glass. If on the rod being further shortened *all* the phenomena suddenly passed into *new* phenomena, the conception of vibration would no longer be serviceable because it would no longer afford us a means of supplementing the new experiences by the previous ones.

When we mentally add to those actions of a human being which we can perceive, sensations and ideas like our own which we cannot perceive, the object of the idea we so form is economical. The idea makes experience intelligible to us; it supplements and supplants experience. This idea is not regarded as a great scientific discovery, only because its formation is so natural that every child conceives it. Now, this is exactly what we do when we imagine a moving body which has just disappeared behind a pillar, or a comet at the moment invisible, as continuing its motion and retaining its previously observed properties. We do this that we may not be surprised by its reappearance. We fill out the gaps in experience by the ideas that experience suggests.

9. Yet not all the prevalent scientific theories originated so naturally and artlessly. Thus, chemical, electrical, and optical phenomena are explained by atoms. But the mental artifice atom was not formed by the principle of continuity; on the contrary, it is a product especially devised for the purpose in view. Atoms

cannot be perceived by the senses; like all substances, they are things of thought. Furthermore, the atoms are invested with properties that absolutely contradict the attributes hitherto observed in bodies. However well fitted atomic theories may be to reproduce certain groups of facts, the physical inquirer who has laid to heart Newton's rules will only admit those theories as *provisional* helps, and will strive to attain, in some more natural way, a satisfactory substitute.

The atomic theory plays a part in physics similar to that of certain auxiliary concepts in mathematics; it is a mathematical *model* for facilitating the mental reproduction of facts. Although we represent vibrations by the harmonic formula, the phenomena of cooling by exponentials, falls by squares of times, etc., no one will fancy that vibrations *in themselves* have anything to do with the circular functions, or the motion of falling bodies with squares. It has simply been observed that the relations between the quantities investigated were similar to certain relations obtaining between familiar mathematical functions, and these *more familiar* ideas are employed as an easy means of supplementing experience. Natural phenomena whose relations are not similar to those of functions with which we are familiar, are at present very difficult to reconstruct. But the progress of mathematics may facilitate the matter.

As mathematical helps of this kind, spaces of more than three dimensions may be used, as I have elsewhere shown. But it is not necessary to regard these, on this account, as anything more than mental artifices.*

This is the case, too, with *all* hypothesis formed for the explanation of new phenomena. Our conceptions of electricity fit in at once with the electrical phenomena, and take almost spontaneously the familiar course, the moment we note that things take place as if attracting and repelling fluids moved on the surface of the conductors. But these mental expedients have nothing whatever to do with the phenomenon *itself*.

* As the outcome of the labors of Lobatchévski, Bolyai, Gauss, and Riemann, the view has gradually obtained currency in the mathematical world, that that which we call *space* is a *particular, actual* case of a more *general*, conceivable case of multiple quantitative manifoldness. The space of sight and touch is a threefold manifoldness; it possesses three dimensions; and every point in it can be defined by three distinct and independent data. But it is possible to conceive of a quadruple or even multiple space-like manifoldness. And the character of the manifoldness may also be differently *conceived* from the manifoldness of actual space. We regard this discovery, which is chiefly due to the labors of Riemann, as a very important one. The properties of actual space are here directly exhibited as objects of *experience*, and the pseudo-theories of geometry that seek to excogitate these properties by metaphysical arguments are overthrown.

A thinking being is supposed to live in the surface of a sphere, with no other kind of space to institute comparisons with. His space will appear to him similarly constituted throughout. He might regard it as infinite, and could only be convinced of the contrary by experience. Starting from any two points of a great circle of the sphere and proceeding at right angles thereto on other great circles, he could hardly expect that the circles last mentioned would intersect. So, also, with respect to the space in which we live, only experience can decide whether it is finite, whether parallel lines intersect in it, or the like. The significance of this elucidation can scarcely be overrated. An enlightenment similar to that which Riemann inaugurated in science was produced in the mind of humanity at large, as regards the surface of the earth, by the discoveries of the first circumnavigators.

The theoretical investigation of the mathematical possibilities above referred to, has, primarily, nothing to do with the question whether things really exist which correspond to these possibilities; and we must not hold mathematicians responsible for the popular absurdities which their investigations have given rise to. The space of sight and touch is *three-dimensional;* that, no one ever yet doubted. If, now, it should be found that bodies vanish from this space, or new bodies get into it, the question might scientifically be discussed whether it would facilitate and promote our insight into things to conceive experiential space as part of a four-dimensional or multi-dimensional space. Yet in such a case, this fourth dimension would, none the less, remain a pure thing of thought, a mental fiction.

But this is not the way matters stand. The phenomena mentioned were not forthcoming until *after* the new views were published, and were then exhibited in the presence of certain persons at spiritualistic *séances*. The fourth

10. My conception of economy of thought was developed out of my experience as a teacher, out of the work of practical instruction. I possessed this conception as early as 1861, when I began my lectures as Privat-Docent, and at the time believed that I was in exclusive possession of the principle—a conviction which will, I think, be found pardonable. I am now, on the contrary, convinced that at least some presentiment of this idea has always, and necessarily must have, been a common possession of all inquirers who have ever made the nature of scientific investigation the subject of their thoughts. The expression of this opinion may assume the most diverse forms; for example, I should most certainly characterize the guiding theme of simplicity and beauty which so distinctly marks the work of Copernicus and Galileo, not only as æsthetical, but also as economical. So, too, Newton's *Regulæ philosophandi* are substantially influenced by economical considerations, although the economical principle as such is not explicitly mentioned. In an interesting article, "An Episode in the History of Philosophy," published in *The Open Court* for April 4, 1895, Mr. Thomas J. McCormack has shown that the idea of the economy of science was very near to the thought of Adam Smith (*Essays*). In recent times the view in question has been repeatedly though diversely expressed, first by myself in my lecture *Ueber die Erhaltung der Arbeit* (1872), then by Clifford in his *Lectures and Essays* (1872), by Kirchhoff in his *Mechanics* (1874), and by Avenarius (1876). To an oral utterance of the political economist A. Herrmann I have already made reference in my *Erhaltung der Arbeit* (p. 55, note 5); but no work by this author treating especially of this subject is known to me.

11. I should also like to make reference here to the supplementary expositions given in my *Popular Scientific Lectures* (English edition, pages 186 et seq.) and in my *Principles of Heat* (German edition, page 294). In the latter work, the criticisms of Petzoldt (*Vierteljahrsschrift für wissenschaftliche Philosophie,* 1891) are considered. Husserl, in the first part of his work, *Logische Untersuchungen* (1900), has recently made some new animadversions on my theory of mental economy; these are in part answered in my reply to Petzoldt. I believe that the best course is to postpone an exhaustive reply until the work of Husserl is completed, and then see whether some understanding cannot be reached. For the present, however, I should

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dimension was a very opportune discovery for the spiritualists and for theologians who were in a quandary about the location of hell. The use the spiritualist makes of the fourth dimension is this. It is possible to move out of a finite straight line, without passing the extremities, through the second dimension; out of a finite closed surface through the third; and, analogously, out of a finite closed space, without passing through the enclosing boundaries, through the fourth dimension. Even the tricks that prestidigitateurs, in the old days, harmlessly executed in three dimensions, are now invested with a new halo by the fourth. But the tricks of the spiritualists, the tying or untying of knots in endless strings, the removing of bodies from closed spaces, are all performed in cases where there is absolutely nothing at stake. All is purposeless jugglery. We have not yet found an *accoucheur* who has accomplished parturition through the fourth dimension. If we should, the question would at once become a serious one. Professor Simony's beautiful tricks in rope-tying, which, as the performance of a prestidigitateur, are very admirable, speak against, not for, the spiritualists.

Everyone is free to set up an opinion and to adduce proofs in support of it. Whether, though, a scientist shall find it worth his while to enter into serious investigations of opinions so advanced, is a question which his reason and instinct alone can decide. If these things, in the end, should turn out to be true, I shall not be ashamed of being the last to believe them. What I have seen of them was not calculated to make me less skeptical.

I myself regarded multi-dimensioned space as a mathematico-physical help even prior to the appearance of Riemann's memoir. But I trust that no one will employ what I have thought, said, and written on this subject as a basis for the fabrication of ghost stories. (Compare Mach, ***Die Geschichte und die Wurzel des Satzes von der Erhaltung der Arbeit.***)

like to premise certain remarks. As a natural inquirer, I am accustomed to begin with some special and definite inquiry, and allow the same to act upon me in all its phases, and to ascend from the special aspects to more general points of view. I followed this custom also in the investigation of the development of physical knowledge. I was obliged to proceed in this manner for the reason that a theory of theory was too difficult a task for me, being doubly difficult in a province in which a minimum of indisputable, general, and independent truths from which everything can be deduced is not furnished at the start, but must first be sought for. An undertaking of this character would doubtless have more prospect of being successful if one took mathematics as one's subject-matter. I accordingly directed my attention to individual phenomena: the adaptation of ideas to facts, the adaptation of ideas to one another,* mental economy, comparison, intellectual experiment, the constancy and continuity of thought, etc. In this inquiry, I found it helpful and restraining to look upon every-day thinking and science in general, as a biological and organic phenomenon, in which logical thinking assumed the position of an ideal limiting case. I do not doubt for a moment that the investigation can be begun at both ends. I have also de-

* *Popular Scientific Lectures*, English edition, pp. 244 et seq., where the adaptation of thoughts to one another is described as the object of theory proper. Grassmann appears to me to say pretty much the same in the introduction to his *Ausdehnungslehre* of 1844, page xix: "The first division of all the sciences is that into real and formal, of which the real sciences depict reality in thought as something independent of thought, and find their truth in the agreement of thought with that reality; the formal sciences, on the other hand, have as their object that which has been posited by thought and itself, find their truth in the agreement of the mental processes with one another."

scribed my efforts as epistemological sketches.* It may be seen from this that I am perfectly able to distinguish between psychological and logical questions, as I believe every one else is who has ever felt the necessity of examining logical processes from the psychological side. But it is doubtful if any one who has read carefully even so much as the logical analysis of Newton's enunciations in my *Mechanics*, will have the temerity to say that I have endeavored to erase all distinctions between the "blind" natural thinking of every-day life and logical thinking. Even if the logical analysis of all the sciences were complete, the biologico-psychological investigation of their development would continue to remain a necessity for me, which would not exclude our making a new logical analysis of this last investigation. If my theory of mental economy be conceived merely as a teleological and provisional theme for guidance, such a conception does not exclude its being based on deeper foundations,† but goes toward making it so. Mental economy is, however, quite apart from this, a very clear logical ideal which retains its value even after its logical analysis has been completed. The systematic form of a science can be deduced from the same principles in many different manners, but some one of these deductions will answer to the principle of economy better than the rest, as I have shown in the case of Gauss's dioptrics.‡ So far as I can now see, I do not think that the investigations of Husserl have affected the results of my inquiries. As for the rest, I

* *Principles of Heat*, Preface to the first German edition.

† *Analysis of the Sensations*, second German edition, pages 64-65.

‡ *Principles of Heat*, German edition, page 394.

must wait until the remainder of his work is published, for which I sincerely wish him the best success.

When I discovered that the idea of mental economy had been so frequently emphasized before and after my enunciation of it, my estimation of my personal achievement was necessarily lowered, but the idea itself appeared to me rather to gain in value on this account; and what appears to Husserl as a degradation of scientific thought, the association of it with vulgar or "blind" thinking, seemed to me to be precisely an exaltation of it. It has outgrown the scholar's study, being deeply rooted in the life of humanity and reacting powerfully upon it.

CHAPTER V.

THE RELATIONS OF MECHANICS TO OTHER DEPARTMENTS OF KNOWLEDGE.

I.

THE RELATIONS OF MECHANICS TO PHYSICS.

1. Purely mechanical phenomena do not exist. The production of mutual accelerations in masses is, to all appearances, a purely dynamical phenomenon. But with these dynamical results are always associated thermal, magnetic, electrical, and chemical phenomena, and the former are always modified in proportion as the latter are asserted. On the other hand, thermal, magnetic, electrical, and chemical conditions also can produce motions. Purely mechanical phenomena, accordingly, are abstractions, made, either intentionally or from necessity, for facilitating our comprehension of things. The same thing is true of the other classes of physical phenomena. Every event belongs, in a strict sense, to all the departments of physics, the latter being separated only by an artificial classification, which is partly conventional, partly physiological, and partly historical.

2. The view that makes mechanics the basis of the remaining branches of physics, and explains all physical phenomena by mechanical ideas, is in our judgment a prejudice. Knowledge which is historically first, is not necessarily the foundation of all that is subsequently